

(D1)

$N\phi = LI$  (definition of inductance)

$L = \frac{N\phi}{I} = \frac{\text{total flux linkage}}{\text{current that generated it}}$

Solenoid self inductance

Mutual Inductance

$L_1 = \frac{\mu_0 N_1^2 A}{l}$

$L_2 = \frac{\mu_0 N_2^2 A}{l}$

$M = \frac{\mu_0 N_1 N_2 A}{l}$

$\Rightarrow L_1 L_2 = M^2$

$\therefore M = \sqrt{L_1 L_2}$

Since coupling is not perfect, introduce  $k$

$M = k \sqrt{L_1 L_2}$

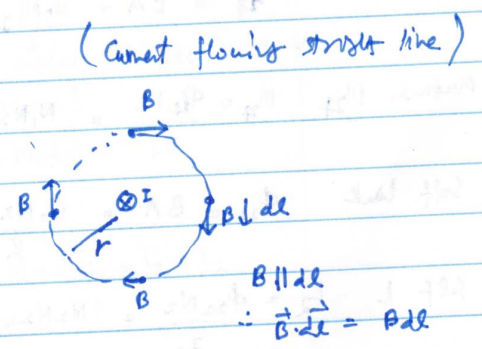
Ampere's Law

$\mu_0 I = \oint \vec{B} \cdot d\vec{l}$

$\mu_0 I = \int B dl$

$\mu_0 I = B(2\pi r)$

$B = \frac{\mu_0 I}{2\pi r}$



(Current flowing in loop)

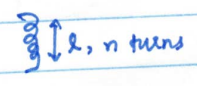
For current flowing in loop you can think of  $2\pi r$   $\otimes$  and sum them to find total



$B = \frac{\mu_0 I}{2\pi r_0} \cdot 2\pi r = \mu_0 I$  at center  $r_0 = r$

For  $N$  turns  $B = \mu_0 NI$

where  $N = \frac{l}{\lambda}$



Now, this  $B$  will create flux linkage

$\phi_B = BA = (\mu_0 NI)(\pi r^2)$

$N\phi_B = N^2 \mu_0 I \pi r^2$

$\frac{N\phi_B}{I} = N^2 \mu_0 \pi r^2$

flux linkage per each turn  
total flux linkage  
self-inductance

$$L - \frac{M^2}{L_2} = L - \frac{k^2 L_1 L_2}{L_2} = 0 \text{ if } k=1$$

$(1-k^2)L$

$k^2 L$

(D2)

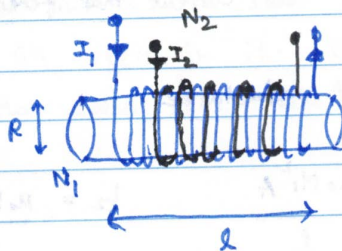
$$\Phi_{21} = BA = \mu_0 N_1 I_1 \cdot A$$

$$\frac{N_2 \Phi_{21}}{I_1} = \mu_0 N_1 N_2 \cdot A$$

Mutual inductance

Summary

Source  $B = \frac{\mu_0 N_1 I_1}{l}$



Self link  $\Phi_{11} = BA = \frac{\mu_0 N_1}{l} \pi R^2 I_1$

Self L  $L_1 = \frac{\Phi_{11} N_1}{I_1} = N_1^2 \frac{\mu_0}{l} \pi R^2$

Mutual link  $\Phi_{21} = BA = \frac{\mu_0 N_1}{l} \pi R^2 I_1$

Mutual  $M_{21} = \frac{\Phi_{21} N_2}{I_1} = N_1 N_2 \frac{\mu_0}{l} \pi R^2$

Self link  $\Phi_{22} = BA = \frac{\mu_0 N_2}{l} \pi R^2 I_2$

Self L  $L_2 = \frac{\Phi_{22} N_2}{I_2} = N_2^2 \frac{\mu_0}{l} \pi R^2$

Mutual link  $\Phi_{12} = BA = \frac{\mu_0 N_2}{l} \pi R^2 I_2$

Mutual  $M_{12} = \frac{\Phi_{12} N_1}{I_2} = N_1 N_2 \frac{\mu_0}{l} \pi R^2$

$$\Rightarrow L_1 = N_1^2 \frac{\mu_0}{l} \pi R^2 \quad L_2 = N_2^2 \frac{\mu_0}{l} \pi R^2 \quad M_{12} = M_{21} = M = N_1 N_2 \frac{\mu_0}{l} \pi R^2$$

$$\rightarrow L_1 L_2 = M^2$$

$$\rightarrow M = \sqrt{L_1 L_2}$$

Introduce imperfect coupling

$$\rightarrow M = k \sqrt{L_1 L_2}$$